ISSN 2239-5393

Two Archaic Mathematical Tablets

Salvatore F. Monaco
(Independent Researcher)

Abstract

Two archaic mathematical tablets show that the "field surveyors' formula", developed for the approximate calculation of the area of fields-plots, had been in use since the fourth millennium B. C. E.

Keywords

Archaic period, field, quadriliteral, mathematical, surveyors.

School texts are well represented in the corpus of archaic tablets. In particular, exercise tablets dealing with field measurements occur since the Uruk IV period[1]. Such exercise texts generally provide the length of the four sides of a quadrilateral, usually in conjunction with the signs $1N_{57}$ (horizontal stroke) and $1N_{58}$ (vertical stroke) in order to identify the opposite sides, with the purpose to give the student-scribe the input data to calculate the area of the field represented by the quadrilateral. True mathematical texts are those which provide, in addition to the input data, the full development of the proposed problems, i.e. the calculations (showing the method adopted to solve the problems), as well as the solutions resulting from the calculations.

The two texts proposed in this paper are the most archaic true mathematical texts ever published, and predate the known ED IIIa mathematical texts (mostly unearthed at Shuruppak) by some five hundred years.

Text no. 1[2]

The tablet (80x66x21mm) is complete but slightly damaged. All the numerical and non-numerical signs in the breaks are easily restored according to the calculations (the

Ricevuto: 01.09.2011. *Accettato*: 15.12.2011.

1 W 19408,76+70b, W 20044,11, 20044,20 and W 20044,33 (attributed to the Uruk IV period) are the oldest school texts dealing with field measurements ever published (ENGLUND – NISSEN 2001). In the first text two exercises are proposed (one on the obverse and one on the reverse of the tablet), for which are given the lengths of the 4 sides of two fields of equivalent area; neither the calculations nor the resulting areas are recorded. The other three tablets give the field's lengths for a single exercise each. For a new published text (CUNES 50-08-076), which proposes four similar exercises on the obverse of the tablet, see presently MONACO 2007.

2 The tablet (CUNES 52-06-031) belongs to the Rosen Collection, now at the Cornell University Museum. It will be published by the A., in photographs, copy, and transliteration, in a forthcoming volume of the CUSAS series

break in case 0101c of the Reverse probably included no signs), or by comparison with similar entries (GAN_2 in case 0102c of the Reverse).

Transliteration

Obverse		Reverse	
0101a	$4N_{34}$ $3N_{14}$ $5N_1$	0101a	$5N_{34}$ $5N_1$
0101b	$\lceil 3N_{34} \rceil$ $2N_{14}$ $6N_1$	0101b	$3N_{34}$ $3N_1$
0101c	$6N_{34}$	0101c	$3N_{50}$ $1N_{14}$ [ ]$^{?}$ GAN_2
0101d	$2N_{34}$ $4N_{14}$		
0102a	$5N_{34}$ $2N_{14}$ $\lceil 4N_1 \rceil$	0102a	$\lceil 5N_{34} \rceil$ $\lceil 4N_{14} \rceil$ $\lceil 2N_1 \rceil$ $2N_8$
0102b	$5N_{34}$ $3N_{14}$	0102b	$\lceil 4N_{34} \rceil$ $2N_1$
0102c	$\lceil 6N_{34} \rceil$	0102c	$4N_{50}$ $\lceil 3N_{14} \rceil$ [GAN_2]
0102d	$2N_{34}$ $3N_{14}$ $\lceil 4N_1 \rceil$		
0103a	$1N_{34}$ $2N_{14}$	0103a	$1N_{34}$ $2N_{14}$
0103b	$1N_{34}$	0103b	$\lceil 5N_{14} \rceil$ $7N_1$
0103c	$1N_{34}$ $2N_{14}$	0103c	$2N_{14}$ $1N_{22}$ $3N_1$ $6N_8$ GAN_2
0103d	$\lceil 5N_{14} \rceil$ $\lceil 4N_1 \rceil$		
0104a	$4N_{14}$	0104a	$\lceil 4N_{14} \rceil$ $5N_1$
0104b	$3N_{14}$	0104b	$3N_{14}$
0104c	$5N_{14}$	0104c	$2N_{22}$ $1N_1$ $5N_8$
0104d	$3N_{14}$	0104d	$2N_{45}$ $2N_{50}$ $9N_{14}$ $1N_{22}$ $\lceil 4N_1 \rceil$ $\lceil 3N_8 \rceil$
0105a	$8N_{34}$	0105a	$8N_{34}$
0105b	$5N_{34}$	0105b	$4N_{34}$ $\lceil 1N_{14} \rceil$ $5N_1$
0105c	$8N_{34}$	0105c	$1N_{45}$ $\lceil 8N_{14} \rceil$ $\lceil GAN_2 \rceil$
0105d	$3N_{34}$ $3N_{14}$		

This text provides in the five rows of the Obverse the input data, i.e. the lengths of the four sides of five quadrilaterals (referred to as GAN_2, i.e. "field"). In each of the five rows of the Reverse are reported the results of the intermediate calculations, executed with the input data of the corresponding row in the Obverse, as well as the resulting area of the quadrilateral. It is known that the ancient scribes, in order to compute the area of a quadrilateral, used the so-called "surveyors' formula", ie:

$$\frac{a+c}{2} * \frac{b+d}{2} = l * h = A$$ semi-sum of lengths (l) times semi-sum of widths (h)[3]

where a, c and b, d are the pairs of opposite sides of the quadrilateral, and A is the resulting area.

[3] This approximate method, used since the IV Mill. BCE for calculating the area of a quadrilateral field, was already analyzed by ALLOTTE DE LA FUŸE 1915. He also provided a formula for computing the difference between the calculated and the actual area of the quadrilateral (*ibid*. 142).

For sake of clarity, in the following table are reported for each exercise the input data measured in nindan (a, b, c, d of the Obverse), the results of the intermediate calculations l and h also measured in nindan (a and b of the Reverse)[4], and the resulting area A of the quadrilateral measured in iku (c of the Reverse)[5]. It shall be noted that a miscalculation was performed for the semi-sum of lengths in the first exercise, reporting 305 nindan, instead of the correct value 317.5 nindan. Consequently, if our hypothesis, that case 0101c of the Reverse did not include any signs other those visible, is correct, then the area was calculated as 558 iku (305 nindan * 183 nindan = 558.15 iku, rounded off to 558 iku), instead of the correct value 581 iku.

	Obverse				Reverse			
	a	b	c	d	a	b	c	d
0101	275	206	360	160	305/317.5	183	558/581	
0102	324	330	360	154	342.2	242	828.2	
0103	80	60	80	54	80	57	45.6!	
0104	40	30	50	30	45	30	13.5	2692.3
0105	480	300	480	210	480	255	1224	
Total calculated area							2669.3	

It is interesting to note that a check-sum was provided in case 0104 of the Reverse of the tablet. Such check-sum was evidently part of the input data for the exercises, since the value of the total area, resulting by summing up the five calculated areas of the quadrilaterals (2669.3 iku), differs from the correct value (2693.3 iku) reported as check-sum, because of the above noted student-scribe miscalculation in the first exercise. Consequently, we should possibly infer that the student-scribe registered on the tablet the value of the given check-sum, without performing the necessary verification, since in such case he would have noted the discrepancy in his totalized area, and detected his mistake in the calculations of the first exercise.

Text no. 2[6]

The tablet, of which only the upper portion remains, probably included more than one exercise (most probably two), since traces of numerical signs are visible in the surviving, but broken initial case of the third row of the Obverse. Also for this text the numerical and non-numerical signs in the breaks were easily restored according to the

[4] The value 305 nindan for the semi-sum of lengths can be the result of a misreading of the measurement of the first side of the quadrilateral as 250 nindan. Anyway, the tablet reads clearly $4N_{34}$ $3N_{14}$ $5N_1$, i.e. 275 nindan. The intermediate calculations were evidently performed on a side tablet, and the results were thereafter copied on the exercise tablet.

[5] Linear measurements are expressed in “nindan”, and area measurements in “iku”.

[6] The tablet, which belongs to a private collection (photographs are available in the CDLI under no. P006413), is the join of two large fragments originally belonging to different tablets. The upper fragment includes in the obverse the mathematical exercise reported in this paper; the reverse is defaced and probably was uninscribed. The lower fragment (cf. photo in CDLI under no. P006397) includes a school exercise dealing with field measurements.

calculations, and to the expected sign (GAN_2) in case O102c. The Reverse of the tablet is completely defaced, and possibly uninscribed.

Transliteration

Obverse		Reverse
0101a	$4N_{34}$ $2N_1$ $1N_{57}$	(*defaced, uninscribed?*)
0101b	$2N_{34}$ $6N_1$ $1N_{58}$	
0101c	$3N_{34}$ $5N_{14}$ $\lceil 4N_1 \rceil$ $1N_{57}$	
0101d	$2N_{34}$ $\lceil 4N_1 \rceil$ $1N_{58}$	
0102a	$3N_{34}$ $5N_{14}$ $\lceil 8N_1 \rceil$ $1N_{57}$	
0102b	$\lceil 2N_{34} \rceil$ $\lceil 5N_1 \rceil$ $[1N_{58}]$	
0102c	$\lceil 1N_{50} \rceil$ [4+] $\lceil 2N_{14} \rceil$ $[1N_{22}]$ $[3N_1]$ $[5N_8]$ $[GAN_2]$	
0103a	[ ] $\lceil 1N_{14} \rceil$ [ ]	
…	…	

The text proposes in the first row the four different lengths of the sides of a quadrilateral, with the two pairs of opposite sides identified, as usual in the field measurement texts, respectively with a horizontal stroke (sign $1N_{57}$) and a vertical stroke (sign $1N_{58}$). In the second row, the first two cases record the result of the intermediate calculations of the mean values (l, h) of the pairs of opposite sides, again identified by the horizontal and vertical strokes, while in the third case is reported the resulting area of the quadrilateral, after multiplication of said mean values, so executing step-by-step the computations of the "surveyors' formula".

In the following table are reported the input data measured in nindan (a, b, c, d of the first row of the Obverse), the results of the intermediate calculations l and h measured in nindan (a and b of the second row of the Obverse), and the resulting area of the quadrilateral A measured in iku (c of the second row of the Obverse).

First row of the Obverse				Second row of the Obverse		
a	b	c	d	a	b	c
242	126	234	124	238	125	297.5

Conclusions

Among the archaic administrative texts there are a number of the so-called "field measurement texts", classifiable in two categories. Those belonging to the first category[7] provide the measurements of the four sides of a plot of field, together with the reference to the owner or the assignee of the plot, which shows that in general the fields were quadrilateral with sides of different lengths. Other texts, belonging to the second

[7] See for instance IM 65109, 65113, 65116, 133955 (no assignee reported), 133957, 133960, MS 4537, which provide the measurements of the four sides of the field, together with the assignee, without the resulting value of the field area

category[8], record the length and width, as well as the area of a plot of field, again together with the reference to the owner or the assignee of the plot. Since the area of such plots of field is equal to length times width, this would imply that such plots were all rectangular in shape. Without excluding that some of the plots could have been actually rectangular, it seems highly probable that the first category of texts registered the actual measurements performed by the "surveyors" in the field, while the measurements reported in the tablets of the second category were already the result of the intermediate calculations for l and h, as shown in the two mathematical texts reported above.

References

ALLOTTE DE LA FUŸE 1915 = F. M. ALLOTTE DE LA FUŸE, "Mesures agraires et formules d'arpentage a l' époque présargonique", *RA* 12, 1915: 117-146.

ATU = Archaic Texts from Uruk.

CDLI = Cuneiform Digital Library Initiative.

CUNES = Museum Numbers of tablets in the Cornell University, Dept. of Near Eastern Studies.

CUSAS = Cornell University Studies in Assyriology and Sumerology, Bethesda, 2007ff.

ENGLUND – GRÉGOIRE 2001 = R. K. ENGLUND – J.-P. GRÉGOIRE, *The Proto-Cuneiform Texts from Jemdet Nasr* (MSVO 1), Berlin 1991.

ENGLUND – NISSEN 2001 = R. K. ENGLUND – H. J. NISSEN, *Archaische Verwaltungstexte aus Uruk: Die Heidelberger Sammlung* (ATU 7), Berlin 1991.

IM = Museum Numbers of tablets in the Iraq Museum, Baghdad.

MONACO 2007 = S. F. MONACO, *The Cornell University Archaic Tablets* (CUSAS 1, no. 7), CDL Press 2007.

MSVO 1, 4-6 = Materialien zu den frühen Schriftzeugnissen des Vorderen Orients, Berlin 1991--.

W = Field Numbers of tablets excavated at Warka.

[8] Cf. MSVO 1, 4-6 from Jemdet Nasr. Other texts provide the measurements of the four sides of the field, together with the assignee, without the resulting value of the field area (see for instance IM 65109, 65113, 65116, 133955 (no assignee reported), 133957, 133960, MS 4537).